Bose-Einstein Condensates in Weak Disorder Potentials

Master thesis

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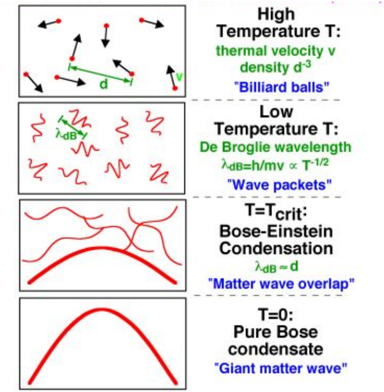
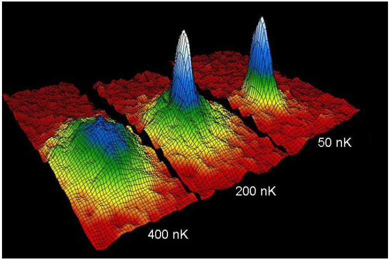
Overview

- Introduction
- Perturbation theory
  - Mean-field approach
  - Condensate density
  - Equation of state
  - Superfluidity
  - Sound velocity
- Model and results
  - Lorentz-correlated disorder and dipolar interaction
- Conclusions and outlook

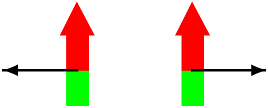
Bose-Einstein condensates





Dipolar interaction

- Atoms: ^{52}Cr, ^{87}Rb
- Molecules: ^{41}K^{87}Rb

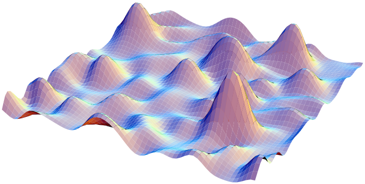


Repulsion

Attraction

Disordered potentials

- Uncontrolled disorder: Wire traps
  Rev. Mod. Phys. **79**, 235 (2007)
- Controlled disorder: Laser speckles
  Nature **453**, 891 (2008)



Perturbation theory

- Gross-Pitaevskii equation for a homogeneous disordered system:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{r}) - \mu + \int d^3\mathbf{r}' V(\mathbf{r}'-\mathbf{r})\psi^*(\mathbf{r}')\psi(\mathbf{r}')\right)\psi(\mathbf{r}) = 0$$

- Statistical properties of the disorder potential: ensemble averages

$$\begin{aligned} \langle U(\mathbf{r})\rangle &= 0 \\ \langle U(\mathbf{r})U(\mathbf{r}')\rangle &= R(\mathbf{r}-\mathbf{r}') \end{aligned}$$

- Perturbative expansion of the solution:

$$\psi(\mathbf{r}) = \psi_0 + \psi_1(\mathbf{r}) + \psi_2(\mathbf{r}) + \ldots$$

Condensate density

- Density matrix: $\rho(\mathbf{r},\mathbf{r}') = \langle\psi(\mathbf{r})\psi(\mathbf{r}')\rangle$
- Fluid density: $n = \left\langle\psi(\mathbf{r})^2\right\rangle$
- Condensate density:

$$n_0 = \lim_{|\mathbf{r}-\mathbf{r}'|\to\infty} \rho(\mathbf{r},\mathbf{r}') = \langle\psi(\mathbf{r})\rangle^2$$

- Condensate depletion due to disorder:

$$n - n_0 = n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{R(\mathbf{k})}{\left[\frac{\hbar^2\mathbf{k}^2}{2m} + 2nV(\mathbf{k})\right]^2} + \ldots$$

Equation of state

- Chemical potential:

$$\mu_b = nV(\mathbf{k}=0) - \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\frac{\hbar^2 k^2}{2m} R(\mathbf{k})}{\left[\frac{\hbar^2 k^2}{2m} + 2nV(\mathbf{k})\right]^2} + \dots$$

- Renormalization, Phys. Rev. A, **84**, 021608(R) (2011)

$$\begin{aligned} \mu(n) &= \mu_b(n) - \mu_b(0) = \mu_b + \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{R(\mathbf{k})}{\frac{\hbar^2 k^2}{2m}} + \dots \\ &= nV(\mathbf{k}=0) + 4n \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{V(\mathbf{k}) R(\mathbf{k}) \left(\frac{\hbar^2 k^2}{2m} + nV(\mathbf{k})\right)}{\frac{\hbar^2 k^2}{2m}\left[\frac{\hbar^2 k^2}{2m} + 2nV(\mathbf{k})\right]^2} + \dots \end{aligned}$$

Superfluidity (1)

- Moving disorder, time-dependent GP equation:

$$\left[-\frac{\hbar^2}{2m}\nabla^2_{\mathbf{r}} + U(\mathbf{r}-\mathbf{v}t) + \int d^3\mathbf{r}' V(\mathbf{r}-\mathbf{r}')\Psi^*_r(\mathbf{r}',t)\Psi_r(\mathbf{r}',t)\right]\Psi_r(\mathbf{r},t) = i\hbar\frac{\partial\Psi_r(\mathbf{r},t)}{\partial t}$$

- Perturbed boosted solution of GP equation:

$$\Psi_r(\mathbf{r},t) = \underbrace{e^{i\mathbf{k}_S\mathbf{r}} e^{-\frac{i}{\hbar}\left(\mu+\frac{\hbar^2 k_S^2}{2m}\right)t}}_{\psi_e} \underbrace{(\psi_0 + \psi_{r1}(\mathbf{r},t) + \ldots)}_{\psi}.$$

- Change of coordinates $\mathbf{x} = \mathbf{r} - \mathbf{v}t$, $\mathbf{K} = \mathbf{k}_S - \frac{m}{\hbar}\mathbf{v}$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - i\frac{\hbar^2}{m}\mathbf{K}\nabla + U(\mathbf{x}) - \mu + \int d^3\mathbf{x}' V(\mathbf{x}-\mathbf{x}')\psi^*(\mathbf{x}')\psi(\mathbf{x}')\right]\psi(\mathbf{x}) = 0$$

Superfluidity (2)

- Derivatives with repect to **K**:

$$\mathbf{p}(\mathbf{x}) = \left(\nabla_{\mathbf{K}} \psi(\mathbf{x})\right)_{\mathbf{K}=0}$$

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + U(\mathbf{x}) - \mu + \int d^3\mathbf{x}' V(\mathbf{x}-\mathbf{x}')n^0(\mathbf{x}')\right] \mathrm{Im}\,\mathbf{p}(\mathbf{x}) = \frac{\hbar^2}{m}\nabla\psi^0(\mathbf{x})$$

- Total fluid wave-vector:

$$\mathbf{k}_{\mathrm{tot}} = \frac{1}{i}\nabla \ln \frac{\Psi}{|\Psi|} = \mathbf{k}_S + \frac{1}{2in}\left(\psi^*\nabla\psi - \psi\nabla\psi^*\right) = \mathbf{k}_S - \hat{D}\mathbf{K}$$

$$\hat{D}(\mathbf{x}) = -\nabla \otimes \frac{\mathrm{Im}\,\mathbf{p}(\mathbf{x})}{\psi^0(\mathbf{x})}$$

Superfluid density

- Energy and momentum densities, arbitrary macroscopic superfluid velocity $\mathbf{k}'_S$

$$\langle n(\mathbf{r})\mathbf{k}_{\mathrm{tot}}(\mathbf{r})\rangle = \hat{n}_S \mathbf{k}'_S + \hat{n}_N \mathbf{k}_v,$$

$$\left\langle n(\mathbf{r})\mathbf{k}^2_{\mathrm{tot}}(\mathbf{r})\right\rangle = \mathbf{k}'_S \hat{n}_S \mathbf{k}'_S + \mathbf{k}_v \hat{n}_N \mathbf{k}_v,$$

- Density of the normal component:

$$\hat{n}_N = 4\psi_0^2 \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\mathbf{k}\otimes\mathbf{k}}{k^2} \frac{R(\mathbf{k})}{\left[\frac{\hbar^2 k^2}{2m} + 2nV(\mathbf{k})\right]^2} + \ldots$$

- Cylindrical symmetry: $\hat{n}_S = \begin{pmatrix} n_{S\rho} & 0 & 0 \\ 0 & n_{S\rho} & 0 \\ 0 & 0 & n_{Sz} \end{pmatrix}$

Sound velocity

- Hydrodynamic equations of averaged quantities:

$$\frac{\partial n_{\mathrm{mac}}(\mathbf{x},t)}{\partial t} + \nabla(\hat{n}_S(\mathbf{x},t)\mathbf{v}'_S(\mathbf{x},t)) = 0$$

$$m\frac{\partial \mathbf{v}'_S(\mathbf{x},t)}{\partial t} + \nabla\left(\frac{m\mathbf{v}'_S(\mathbf{x},t)^2}{2} + \mu(n_{\mathrm{mac}}(\mathbf{x},t))\right) = 0$$

- Small variation from the equilibrium:

$$c_{\mathbf{q}}^2 = \frac{1}{m}\frac{\partial \mu}{\partial n}\mathbf{q}^T\hat{n}_S\mathbf{q}$$

- Measurable by Bragg spectroscopy

Anisotropic disorder and dipolar interaction

- Modeling disorder and interaction:

$$R(\mathbf{k}) = \frac{R}{1+\sigma_\rho^2 k_\rho^2 + \sigma_z^2 k_z^2}, \; V(\mathbf{k}) = g + \frac{C_d}{3}\left(3\cos^2\phi(\mathbf{m},\mathbf{k}) - 1\right)$$

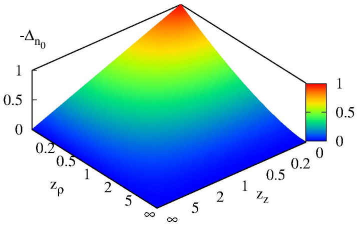
- Corrections are expressed as:

$$\Delta_A = \lim_{R\to 0} \frac{\frac{A}{A_0} - A_d}{\frac{n_{\mathrm{HM}}}{n}}, \; n_{\mathrm{HM}} = \frac{m^{\frac{3}{2}} R\sqrt{n}}{4\pi\hbar^3\sqrt{g}}$$

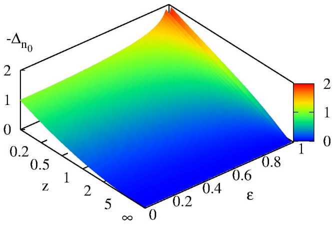
Phys. Rev. Lett. **69**, 644 (1992)

Condensate depletion

$\epsilon = 0$

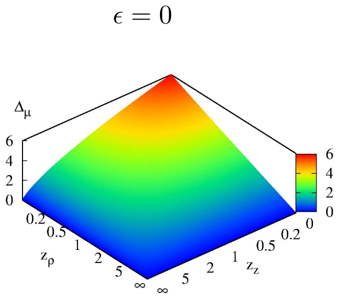


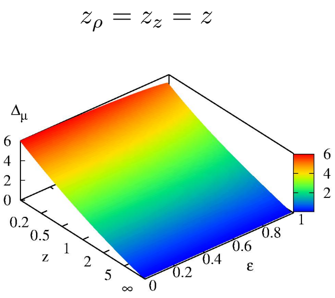
$z_\rho = z_z = z$



$$z = \frac{\sqrt{2}\sigma}{\xi},\ \xi = \frac{\hbar}{2\sqrt{mng}}\ \text{(healing length)}\ ,\ \epsilon = \frac{C_d}{3g}$$

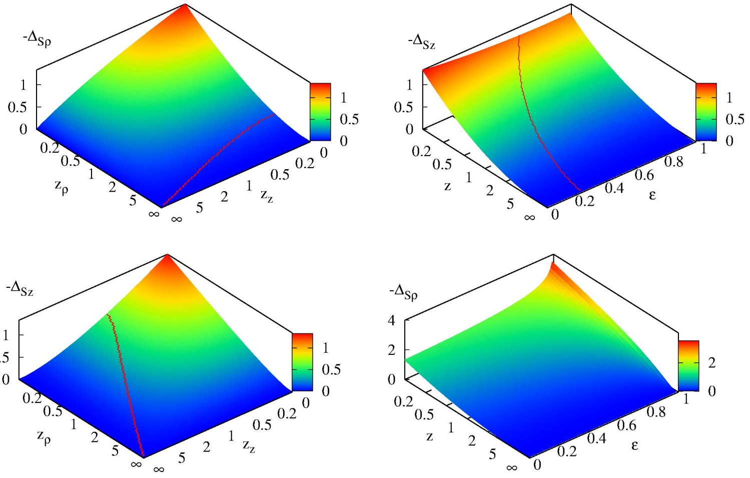
Equation of state



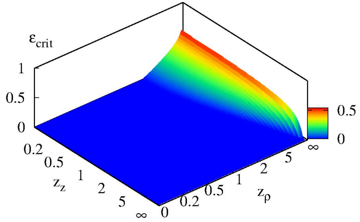


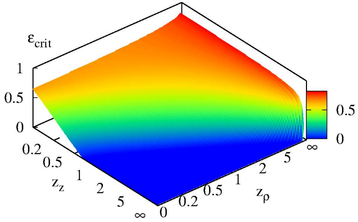
Superfluid depletion

$\epsilon = 0$ $\qquad\qquad$ $z_\rho = z_z = z$

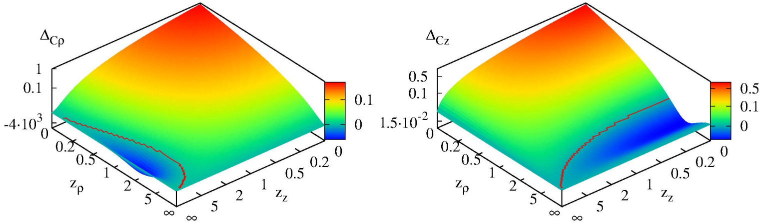


Critical values of ϵ



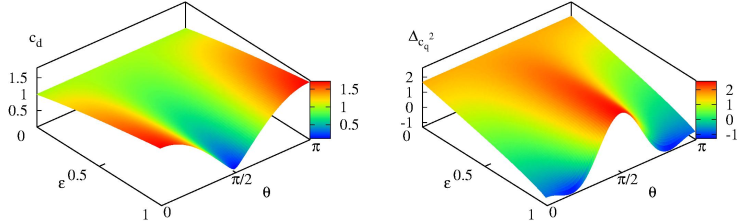


Sound velocity for $\epsilon = 0$



$$c_{\mathbf{q}}^2 = c_\rho^2 \sin^2 \phi(\mathbf{q}, \mathbf{e}_z) + c_z^2 \cos^2 \phi(\mathbf{q}, \mathbf{e}_z)$$

Sound velocity for $z_\rho = z_z = 0$



$$c_{\mathbf{q}}^2 = c_0^2 \left(c_d(\mathbf{q})^2 + \frac{n_{\mathrm{HM}}}{n} \Delta_{c_{\mathbf{q}}^2} \right), \ \theta = \phi(\mathbf{q}, \mathbf{e}_z)$$

Conclusions and outlook

- Summary
  - Consistent mean-field model
  - Interesting anisotropic effects
    - Condensate depletion
    - Superfluid depletion
    - Sound velocity
  - All results are measurable
- Further research
  - Automatization of higher order calculation
  - Numeric simulations for correlation function of laser speckles
  - Superfluid definition for finite temperatures:
    - Change of T_c and possible new phases

Appendix: Spatial average

- Assumption: Spatial averaging over large volumes coincides with the disorder average.
- Order parameter

$$\begin{aligned} \langle\psi(\mathbf{r})\psi(\mathbf{r}')\rangle &\approx \frac{1}{V_0^2}\int_{V_0} d^3\mathbf{r}_1 d^3\mathbf{r}_2 \,\langle\psi(\mathbf{r}+\mathbf{r}_1)\psi(\mathbf{r}'+\mathbf{r}_2)\rangle \\ &= \langle\langle\psi(\mathbf{r})\rangle\,\langle\psi(\mathbf{r}')\rangle\rangle = \langle\psi(\mathbf{r})\rangle^2 \end{aligned}$$

- Macroscopic hydrodynamic equations should be independent on the microscopic realization, therefore they relate local-spatially averaged quantities. The above assumption justifies the equations.