Quantum disordering of the 111 state and the compressible-incompressible transition in quantum Hall bilayer systems

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We systematically discuss properties of quantum disordered states of the quantum Hall bilayer at $\nu_T=1$. For one of them, the so-called vortex metal state, we find off-diagonal long-range order of *algebraic* kind, and derive its transport properties. It is shown that this state is relevant for the explanation of the "imperfect" superfluid behavior and persistent intercorrelations, for large distances between layers, that were found in experiments.

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I. INTRODUCTION

Electrons in quantum Hall bilayer systems at total filling factor $\nu_T=1$ naturally correlate in two different ways due to Pauli principle and Coulomb interaction. If the layers are sufficiently far apart, dominant correlations would be those of intralayer kind because electrons in one layer are unable to sense what is taking place in the opposite layer. This does not hold, however, in the limit of small layer separation. Instead, with decreasing d/l_B, the ratio of the distance between layers to the magnetic length, the correlations between electrons in different layers gain strength and begin to compete with intralayer correlations. It is the interplay of those two kinds of correlations that we focus on in this paper.

For the case of prevalent interlayer correlations, there are already a few theoretical models at hand which provide a satisfactory description: 111 state given by Halperin's[1] 111 wave function $\Psi_{111}=\Pi_{i<j}(z_{i\uparrow}-z_{j\uparrow})\Pi_{k<l}(z_{k\downarrow}-z_{l\downarrow})\Pi_{m,n}(z_{m\uparrow}-z_{n\downarrow})$, quantum Hall ferromagnet,[2] condensate of excitons,[3] or composite bosons.[4] Nevertheless, both theoretically and experimentally, it is evident that with increasing d/l_B, a quantum disordering of this state is bound to take place. For example, the tunneling peak observed by Spielman *et al.*[5] is indeed sharp and pronounced, but its nature is more that of a resonance than of the speculated Josephson effect, while the temperature dependences of Hall and longitudinal resistances in experiments of Kellogg *et al.*[6] and Tutuc *et al.*[7] do not provide support to the predicted Berezinskii-Kosterlitz-Thouless (BKT) scenario of a bilayer finite temperature phase transition.[2] Deeper understanding of the regime $d\sim l_B$ is therefore an important, open problem in the physics of quantum Hall bilayers and strongly correlated electron systems in general.

Hereinafter, we present some results which pertain to quantum disordering that is believed to take place in the quantum Hall bilayer at $\nu_T=1$. The ground state at $d=0$ is a Bose condensate well described by 111 wave function due to Halperin, while the low-lying excitations are composite bosons, i.e., electrons dressed with one quantum of magnetic flux.[4] The idea of disordering that we employ is to allow the formation of composite fermions (i.e., electrons dressed with *two* quanta of magnetic flux) that coexist with composite bosons.[8] There are two ways to introduce composite fermions into the Bose condensate and this will be explained in Sec. II. We then pursue a phenomenological Chern-Simons transport theory of Drude in order to examine the elementary predictions of those two model states. In Sec. III, we arrive at an effective gauge theory for both cases. This enables us to calculate the correlation functions, modes of low-lying excitations, and characteristic off-diagonal long-range order (ODLRO). We will be primarily interested in the pseudospin channel of these states. In one of those, the so-called vortex metal state that we believe may appear in the bilayer at larger d/l_B as a manifestation of increasing intracorrelations, we derive an *algebraic* ODLRO. In Sec. IV, we focus on the incompressible region and the crossover around the critical layer separation. We will argue that our field-theoretical, homogeneous picture in fact suggests that vortex metal, if relevant for the strongly coupled, incompressible region, may appear only localized in the form of islands in the background of the superfluid state for smaller d/l_B. In Sec. V, we give a more thorough analysis of the experiments on bilayer, addressing especially the compressible, weakly coupled region, and the question of persistent intercorrelations[9] in the framework of the vortex metal state. Section VI is devoted to discussion and conclusion. For the sake of clarity and in order to make the text self-contained, some of the known results[8,10] will be rederived in this paper.

II. TRIAL WAVE FUNCTIONS FOR THE BILAYER

Building on Laughlin's proposal for the wave function of a single quantum Hall layer,[11] the construction of Rezayi-Read wave function[12] for $\nu=1/2$ and Halperin's 111 wave function for bilayer,[1] we may formally imagine that there are two species of electrons in each layer (z,w), which are all mutually correlated through intracorrelations (within the same layer) and intercorrelations (between opposite layers) (Fig. 1).

Starting from the 111 function of the Bose condensate, we will minimally deform it in order to include the composite fermions. Given that each particle binds the same number of flux quanta and taking Pauli principle into account, this becomes a combinatorial problem with two solutions. In the first case,

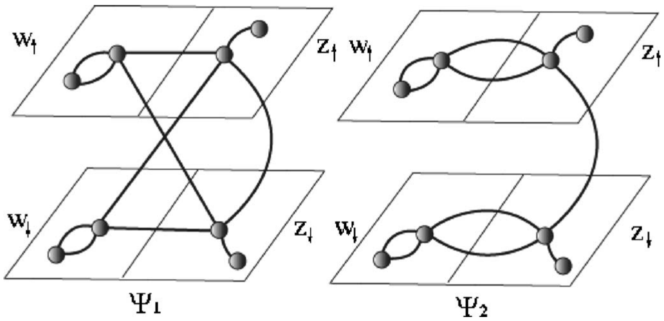

FIG. 1. Correlations between electrons in two layers.

$$\Psi_1 = \mathcal{PA}\Bigg[\prod_{i<j}(z_{i\uparrow}-z_{j\uparrow})\prod_{k<l}(z_{k\downarrow}-z_{l\downarrow})\prod_{p,q}(z_{p\uparrow}-z_{q\downarrow}) \times \Phi_f(w_\uparrow,\bar{w}_\uparrow)\prod_{i<j}(w_{i\uparrow}-w_{j\uparrow})^2 \times \Phi_f(w_\downarrow,\bar{w}_\downarrow)\prod_{k<l}(w_{k\downarrow}-w_{l\downarrow})^2 \times \prod_{i,j}(z_{i\uparrow}-w_{j\uparrow})\prod_{k,l}(z_{k\uparrow}-w_{l\downarrow}) \times \prod_{p,q}(z_{p,\downarrow}-w_{q,\uparrow})\prod_{m,n}(z_{m\downarrow}-w_{n\downarrow})\Bigg]. \quad (1)$$

The first line in this formula can be recognized as 111 function, followed by two $\nu=1/2$ separate layers (Φ_f's denote the Slater determinants of free composite fermions), while the last two lines stem from the flux-particle constraint (all these correlations are depicted on the left hand side of Fig. 1). $\mathcal{P}$ and $\mathcal{A}$ denote projection to the lowest Landau level (LLL) and fermionic antisymmetrization (independently for each layer), respectively. In the thermodynamic limit, the relation between the number of particles and flux quanta reads[10]

$$N_\Phi = N_{b\uparrow}+N_{b\downarrow}+N_{f\uparrow}+N_{f\downarrow} = 2N_{f\uparrow}+N_{b\uparrow}+N_{b\downarrow} = 2N_{f\downarrow}+N_{b\uparrow}+N_{b\downarrow}. \quad (2)$$

N_Φ is the number of flux quanta through the system and $N_{b\sigma}$ and $N_{f\sigma}$ are the number of bosons and fermions inside the layer σ, respectively; $\sigma=\uparrow,\downarrow$ is the layer index. Equation (2) enforces an additional constraint $N_{f\uparrow}=N_{f\downarrow}$. Therefore, the number of fermions is balanced in two layers, while the boson numbers are not subject to any such constraint. This fact is important because of the broken symmetry of spontaneous interlayer phase coherence in the 111 state, which demands nonconservation of $N_{b\uparrow}-N_{b\downarrow}$. Although we work in a fixed (relative) number representation (allowed in a broken symmetry case) to account for a broken symmetry situation, we need to have a possibility of unconstrained relative number of bosons. Then, a superposition of the wave functions of the form in Eq. (1) would lead to the usual representation.

In the second case which is expected to describe dominant intracorrelations, fermions bind exclusively within the layer they belong to (right side of Fig. 1) and the corresponding wave function is

$$\Psi_2 = \mathcal{PA}\Bigg[\prod_{i<j}(z_{i\uparrow}-z_{j\uparrow})\prod_{k<l}(z_{k\downarrow}-z_{l\downarrow})\prod_{p,q}(z_{p\uparrow}-z_{q\downarrow}) \times \Phi_f(w_\uparrow,\bar{w}_\uparrow)\prod_{i<j}(w_{i\uparrow}-w_{j\uparrow})^2 \times \Phi_f(w_\downarrow,\bar{w}_\downarrow)\prod_{k<l}(w_{k\downarrow}-w_{l\downarrow})^2 \times \prod_{i,j}(z_{i\uparrow}-w_{j\uparrow})^2\prod_{k,l}(z_{k\downarrow}-w_{l\downarrow})^2\Bigg]. \quad (3)$$

In this case, the flux-particle relation[10] is

$$N_\Phi = 2N_{f\uparrow}+2N_{b\uparrow} = 2N_{f\downarrow}+2N_{b\downarrow} = 2N_{f\uparrow}+N_{b\uparrow}+N_{b\downarrow} = 2N_{f\downarrow}+N_{b\uparrow}+N_{b\downarrow}, \quad (4)$$

implying that both fermion and boson numbers must be balanced: $N_{f\uparrow}=N_{f\downarrow}$ and $N_{b\uparrow}=N_{b\downarrow}$.

In Ref. 8, the authors numerically calculated the overlap of Ψ_1 with the exact ground-state wave function for a system of five electrons in each layer with varying d/l_B. Their results seem to demonstrate convincingly that (at least for small systems) the approach with trial wave functions that interpolate between two well-established limits, namely, those of 111 state and decoupled $\nu=1/2$ layers, is not only an artificial mathematical construction but also corresponds to physical reality. Despite the fact that the number of electrons in this simulation is certainly well below the thermodynamic limit, the fact that the overlaps between Ψ_1 and the exact ground state display peaks very close to 1 at small d/l_B provides confidence in the choice of wave function Ψ_1 (at least for small d/l_B).

If there is a phase separation in between the sea of composite bosons and composite fermions, the phase transition will be of the first order. Such a scenario is launched in Ref. 13, where the authors imagine static, isolated regions of incoherent phase inside 111 phase. Although this model correctly explains some of the observed phenomena (e.g., semicircle law), the persistence of intercorrelations in the weakly coupled, compressible regime[9] which gradually die out suggests a continuous transition. Such a possibility is naturally present in the picture of composite boson-composite fermion mixture.

A transport theory of Drude kind can be easily formulated[8] if we consider that composite fermions bind two quanta of magnetic flux, unlike composite bosons which bind only one quantum of magnetic flux. As long as we remain in the random-phase approximation (RPA), they can all be treated as free particles moving in the presence of the effective field which is given by the sum of the external and self-consistently induced electric field. In the first case (Ψ_1), the effective field as seen by particles in the layer σ is

$$\mathcal{E}_f^\sigma = \mathbf{E}^\sigma - 2\epsilon\mathbf{J}_f^\sigma - \epsilon(\mathbf{J}_b^1+\mathbf{J}_b^2), \quad (5)$$

$$\mathcal{E}_b^\sigma = \mathbf{E}^\sigma - \epsilon(\mathbf{J}_b^1+\mathbf{J}_b^2+\mathbf{J}_f^1+\mathbf{J}_f^2), \quad (6)$$

where $\mathbf{J}_{f(b)}^\sigma$ denote Fermi and Bose currents in the layer σ and

$$\epsilon = \begin{bmatrix} 0 & \delta \\ -\delta & 0 \end{bmatrix},$$

with $\delta = \frac{h}{e^2}$. Transport equations are

$$\mathcal{E}^{\sigma}_{f(b)} = \rho^{\sigma}_{f(b)} \mathbf{J}^{\sigma}_{f(b)} \tag{7}$$

and, as required by symmetry, $\rho^1_{f(b)} = \rho^2_{f(b)}$, while the total current is given by $\mathbf{J}^{\sigma} = \mathbf{J}^{\sigma}_b + \mathbf{J}^{\sigma}_f$. We define single layer resistance (ρ^{11}) and drag resistance (ρ^D) as follows:

$$\mathbf{E}^1 = \rho^{11} \mathbf{J}^1, \tag{8}$$

$$\mathbf{E}^2 = \rho^D \mathbf{J}^1. \tag{9}$$

When both layers have the same filling, $\nu_1 = \nu_2 = 1/2$, tensors ρ_b and ρ_f are diagonal (because the composite particles are in zero net field): $\rho_b = \mathrm{diag}[\rho_{bxx}, \rho_{bxx}]$ and $\rho_f = \mathrm{diag}[\rho_{fxx}, \rho_{fxx}]$, and in the case of drag, we have in addition $\mathbf{J}^2 = 0; \mathbf{J}^1$ is finite. Then, from Eqs. (5)–(9) via elementary algebraic manipulations, we obtain

$$\rho^{11} = 1/2\{(\rho_b^{-1} + \rho_f^{-1})^{-1} + 2\epsilon + [(\rho_f + 2\epsilon)^{-1} + \rho_b^{-1}]^{-1}\}, \tag{10}$$

$$\rho^{D} = 1/2\{(\rho_b^{-1} + \rho_f^{-1})^{-1} + 2\epsilon - [(\rho_f + 2\epsilon)^{-1} + \rho_b^{-1}]^{-1}\}, \tag{11}$$

or in terms of matrix elements,

$$\rho^D_{xx} = -\frac{2\rho^2_{bxx}\delta^2}{(\rho_{bxx} + \rho_{fxx})^3 + 4(\rho_{bxx} + \rho_{fxx})\delta^2}, \tag{12}$$

$$\rho^D_{xy} = \frac{\delta(2\rho_{bxx}\rho_{fxx} + \rho^2_{fxx} + 4\delta^2)}{\rho^2_{bxx} + 2\rho_{bxx}\rho_{fxx} + \rho^2_{fxx} + 4\delta^2}, \tag{13}$$

$$\rho^{11}_{xx} = \frac{2\rho^2_{bxx}\delta^2}{(\rho_{bxx} + \rho_{fxx})^3 + 4(\rho_{bxx} + \rho_{fxx})\delta^2} + \frac{\rho_{bxx}\rho_{fxx}}{(\rho_{bxx} + \rho_{fxx})}. \tag{14}$$

The formulas in Eqs. (12)–(14) include parameters δ, ρ_{bxx}, and ρ_{fxx}, the last two being the free parameters about which nothing can be said *a priori*. This prompted Simon *et al.*[8] to reason as follows. At large d/l_B, the number of composite bosons is small because the condensate is broken and ρ_{bxx} is large compared to δ, which is the typical Hall resistance. On the other hand, from the experiments,[14] we know that for large d/l_B holds $\rho_{fxx} \ll \delta$. Furthermore, even as d/l_B is decreased, we expect ρ_{fxx} to increase only slightly.[8] All in all, for large d/l_B, they assume $\rho_{bxx} \gg \delta \gg \rho_{fxx}$, and if in addition we allow $\rho_{bxx}\rho_{fxx} \ll \delta^2$, asymptotically we obtain

$$\rho^D_{xx} \approx -\frac{2\delta^2}{\rho_{bxx}}, \tag{15}$$

$$\rho^D_{xy} \approx 4\delta\left(\frac{\delta}{\rho_{bxx}}\right)^2, \tag{16}$$

$$|\rho^{11}_{xx}| \approx |\rho^D_{xx}|. \tag{17}$$

Semicircle law follows directly from the previous formulas,

$$(\rho^D_{xx})^2 + \left(\rho^D_{xy} - \frac{\delta}{2}\right)^2 \approx \left(\frac{\delta}{2}\right)^2, \tag{18}$$

in agreement with Ref. 13 (semicircle law is of general validity for two-component systems in two dimensions and it serves us as a crucial test for the line of reasoning quoted above, which may at first sound somewhat naive).

In the opposite limit (when d/l_B is reduced), $\rho_{bxx} \ll \rho_{fxx} \ll \delta$ because ρ_{bxx} drops as a result of Bose condensation.[8] When $\rho_{bxx} \to 0$, we obtain the quantization of Coulomb drag,

$$\rho^D_{xy} \approx \delta, \tag{19}$$

$$\rho^D_{xx} \to 0, \tag{20}$$

as measured by Kellogg *et al.*[6]

Let us return now to the case of dominant intracorrelations, the vortex metal state[10] represented by Eq. (3). From Fig. 1, the formulas for effective fields are modified into

$$\mathcal{E}^{\sigma}_f = \mathbf{E}^{\sigma} - 2\epsilon\mathbf{J}^{\sigma}_f - 2\epsilon\mathbf{J}^{\sigma}_b, \tag{21}$$

$$\mathcal{E}^{\sigma}_b = \mathbf{E}^{\sigma} - \epsilon(\mathbf{J}^1_b + \mathbf{J}^2_b + 2\mathbf{J}^{\sigma}_f), \tag{22}$$

and the analogous calculation yields the resistivity tensors,

$$\rho^{11} = \frac{1}{2}\{(\rho_b^{-1} + \rho_f^{-1})^{-1} + 2\epsilon + [(\rho_b - 2\epsilon)^{-1} + \rho_f^{-1}]^{-1} \times [(\rho_b - 2\epsilon)^{-1}\rho_b + 2\rho_f^{-1}\epsilon]\}, \tag{23}$$

$$\rho^{D} = \frac{1}{2}\{(\rho_b^{-1} + \rho_f^{-1})^{-1} + 2\epsilon - [(\rho_b - 2\epsilon)^{-1} + \rho_f^{-1}]^{-1} \times [(\rho_b - 2\epsilon)^{-1}\rho_b + 2\rho_f^{-1}\epsilon]\}. \tag{24}$$

The matrix elements of these tensors are

$$\rho^D_{xx} = -\frac{2\rho^2_{fxx}\delta^2}{(\rho_{bxx} + \rho_{fxx})^3 + 4(\rho_{bxx} + \rho_{fxx})\delta^2}, \tag{25}$$

$$\rho^D_{xy} = \frac{\rho^2_{fxx}\delta}{(\rho_{fxx} + \rho_{bxx})^2 + 4\delta^2}, \tag{26}$$

$$\rho^{11}_{xx} = \frac{2\rho^2_{fxx}\delta^2}{(\rho_{bxx} + \rho_{fxx})^3 + 4(\rho_{bxx} + \rho_{fxx})\delta^2} + \frac{\rho_{bxx}\rho_{fxx}}{(\rho_{bxx} + \rho_{fxx})}. \tag{27}$$

In this case as well, there are two physically significant limits depending on the assumptions for the values of ρ_{bxx} and ρ_{fxx}. In the case when $\rho_{bxx} \ll \rho_{fxx} \ll \delta$,

$$\rho^D_{xx} \approx -\frac{\rho_{fxx}}{2}, \tag{28}$$

$$\rho^D_{xy} \approx \frac{1}{4}\frac{\rho^2_{fxx}}{\delta}, \tag{29}$$

$$\rho_{xx}^{11} \approx \frac{\rho_{fxx}}{2}, \tag{30}$$

and the semicircle law follows [Eq. (18)], whereas $|\rho_{xx}^{D}| = |\rho_{xx}^{11}|$. Similarly, in the regime $\rho_{bxx} \ll \delta \ll \rho_{fxx}$, we deduce the quantization of Coulomb drag [Eqs. (19) and (20)].

We emphasize that these two limits are different from those in Simon *et al.*[8] For example, the semicircle law was derived assuming that ρ_{bxx} is small (which is exactly the opposite situation to the one in Ref. 8), while ρ_{fxx} is not necessarily small with respect to δ. As noted in the first case above, the exact values for ρ_{bxx} and ρ_{fxx} are in fact unknown and this prevents us from discriminating between the different proposed limits. In other words, we cannot say which one of the proposed limits is plausible—the analysis above serves us only to conclude that each of the two composite boson-composite fermion mixed states is able (with certain assumptions) to reproduce the phenomenology of drag experiments.

III. CHERN-SIMONS THEORY FOR BILAYER

Encouraged by the preliminary analysis from the previous section, we will pursue the idea of composite boson-composite fermion mixture further by formulating an example of Chern-Simons (CS) field theory which can contain wave functions Ψ_1, and Ψ_2 as ground states. We do not embark on such a task only for the sake of completeness, but also because such a theory would enable efficient calculation of response functions and provide insight into the long-range order of the system and the nature of low-lying excitations. A general drawback of CS theories is the inability to include the projection to LLL which is the arena where all the physics must be taking place. Nevertheless, we will use these theories established in the works of Zhang *et al.*[15] for composite bosons and Halperin *et al.*[16] for composite fermions because even projected to the LLL type of theories, of Murthy and Shankar,[17] came to the conclusion that in order to get, in the most efficient way, to the qualitative picture of the physics of response, the usual CS theories are quite enough and accurate. In addition to this simplification, in constructing the CS theories, we will neglect the antisymmetrization requirement implied by Eqs. (1) and (3). The reason for this is that just like in hierarchical constructions, composite fermions represent meron excitations (see Ref. 10) that quantum disorder the 111 state, and, as it is usual when we discuss the dual picture of the fractional quantum Hall effect,[18] we do not extend the antisymmetrization requirement to the quasiparticle part of the electron fluid.

Therefore, we start from the Lagrangian given by[10]

$$\mathcal{L} = \sum_{\sigma} \left[\Psi_{\sigma}^{\dagger}(i\partial_0 - a_0^{F\sigma} + A_0 + \sigma B_0)\Psi_{\sigma} - \frac{1}{2m}|(-i\nabla + \mathbf{a}^{F\sigma} - \mathbf{A} - \sigma\mathbf{B})\Psi_{\sigma}|^2 \right] + \sum_{\sigma} \left[\Phi_{\sigma}^{\dagger}(i\partial_0 - a_0^{B\sigma} + A_0 + \sigma B_0)\Phi_{\sigma} - \frac{1}{2m}|(-i\nabla + \mathbf{a}^{B\sigma} - \mathbf{A} - \sigma\mathbf{B})\Phi_{\sigma}|^2 \right] + \sum_{\sigma} \sum_{i=F,B} \frac{1}{2\pi}\frac{1}{2} a_0^{i\sigma} \times (\nabla \times \tilde{\mathbf{a}}^{i\sigma}) - \frac{1}{2}\sum_{\sigma,\sigma'} \int d^2\mathbf{r}'\, \delta\rho_{\sigma}(\mathbf{r}) V_{\sigma\sigma'} \delta\rho_{\sigma'}(\mathbf{r}'), \tag{31}$$

where σ enumerates the layers, Ψ_{σ} and Φ_{σ} are composite fermion and composite boson fields in the layer σ, $V_{\uparrow\uparrow} = V_{\downarrow\downarrow} \equiv V_a$, $V_{\uparrow\downarrow} = V_{\downarrow\uparrow} \equiv V_e$, and the densities are $\delta\rho_{\sigma} = \delta\rho_{\sigma}^{F} + \delta\rho_{\sigma}^{B}$. By $\mathbf{A}$ (and $\mathbf{B}$) here, we mean external fields in addition to the vector potential of the uniform magnetic field $\mathbf{A_B}$, which is accounted for and included in gauge fields $\mathbf{a}^{F(B)\sigma}$. Therefore, we have $\mathbf{a}^{F(B)\sigma} = \tilde{\mathbf{a}}^{F(B)\sigma} - \mathbf{A_B}$. External fields $\mathbf{A}$ and $\mathbf{B}$ couple with charge and pseudospin, and in general we must introduce four gauge fields $\mathbf{a}^{F(B)\sigma}$. Fortunately, not all of them are independent. In the first case, the relation analogous to Eq. (2) becomes the following gauge field equation:

$$\frac{1}{2\pi}\nabla \times \mathbf{a}^{F\sigma} = 2\delta\rho^{F\sigma} + \delta\rho^{B\uparrow} + \delta\rho^{B\downarrow},$$

$$\frac{1}{2\pi}\nabla \times \mathbf{a}^{B\sigma} = \delta\rho^{F\uparrow} + \delta\rho^{F\downarrow} + \delta\rho^{B\uparrow} + \delta\rho^{B\downarrow}. \tag{32}$$

From the equations above, it is obvious that there are only two linearly independent gauge fields: $\mathbf{a}_C = \frac{\mathbf{a}^{F\uparrow} + \mathbf{a}^{F\downarrow}}{2} = \frac{\mathbf{a}^{B\uparrow} + \mathbf{a}^{B\downarrow}}{2}$ and $\mathbf{a}_S = \frac{\mathbf{a}^{F\uparrow} - \mathbf{a}^{F\downarrow}}{2}$, and Eq. (32) expressed in Coulomb gauge reads $\frac{ika_C}{2\pi} = \delta\rho_{\uparrow} + \delta\rho_{\downarrow} \equiv \delta\rho$ and $\frac{ika_S}{2\pi} = \delta\rho^{F\uparrow} - \delta\rho^{F\downarrow} \equiv \delta\rho_S^F$ (a_C and a_S are the transverse components of the gauge fields). These are the constraints we wish to include into the functional integral via Lagrange multipliers a_0^C and a_0^S. The interaction part of the Lagrangian is easily diagonalized by introducing $V_C = \frac{V_a + V_e}{2}$ and $V_S = \frac{V_a - V_e}{2}$.

The strategy for integrating out the bosonic functions is the Madelung ansatz $\phi_{\sigma} = \sqrt{\rho_{\sigma} + \bar{\rho}_{\sigma}} e^{i\theta_{\sigma}}$, which expands the wave function in terms of a product of its amplitude and phase factor, while fermionic functions are treated, as elaborated in Ref. 16. After Fourier transformation, within the quadratic (RPA) approximation, and introducing substitutions $\delta\rho_C^i = \delta\rho_{\uparrow}^i + \delta\rho_{\downarrow}^i$ and $\delta\rho_S^i = \delta\rho_{\uparrow}^i - \delta\rho_{\downarrow}^i$, $i = F$, B and $\theta_C = \frac{\theta_{\uparrow} + \theta_{\downarrow}}{2}$, $\theta_S = \frac{\theta_{\uparrow} - \theta_{\downarrow}}{2}$, all the terms neatly decouple into a charge and a pseudospin channel,

$$\mathcal{L}_C = K_{00}(\delta a_0^C)^2 + K_{11}(\delta a_C)^2 + i\omega\delta\rho_C^B\theta_C - \delta\rho_C^B\delta a_0^C - \frac{\bar{\rho}_b}{m}k^2\theta_C^2 - \frac{\bar{\rho}_b}{m}(\delta a_C)^2 + \frac{1}{2\pi}a_0^C ika_C - \frac{1}{2}\frac{k^2 a_C^2}{(2\pi)^2}V_C, \tag{33}$$

$$\mathcal{L}_{PS} = K_{00}(\delta a_0^S)^2 + K_{11}(\delta a_S)^2 + i\omega\delta\rho_S^B\theta_S - \delta\rho_S B_0 - \frac{\bar{\rho}_b}{m}k^2\theta_S^2 - \frac{\bar{\rho}_b}{m}B^2 + \frac{1}{2\pi}a_0^S ika_S - \frac{1}{2}V_S\left(\delta\rho_S^B + \frac{ik}{2\pi}a_S\right)^2, \tag{34}$$

where $\delta a_0^C \equiv a_0^C - A_0$, $\delta a_C \equiv a_C - A$, $\delta a_0^S \equiv a_0^S - B_0$, $\delta a_S \equiv a_S - B$, and $\bar{\rho}_b$ the mean density of bosons in (each) layer. In writing down Eqs. (33) and (34), we utilize a compact notation suppressing k $(-k)$ dependence, where all the quadratic terms of

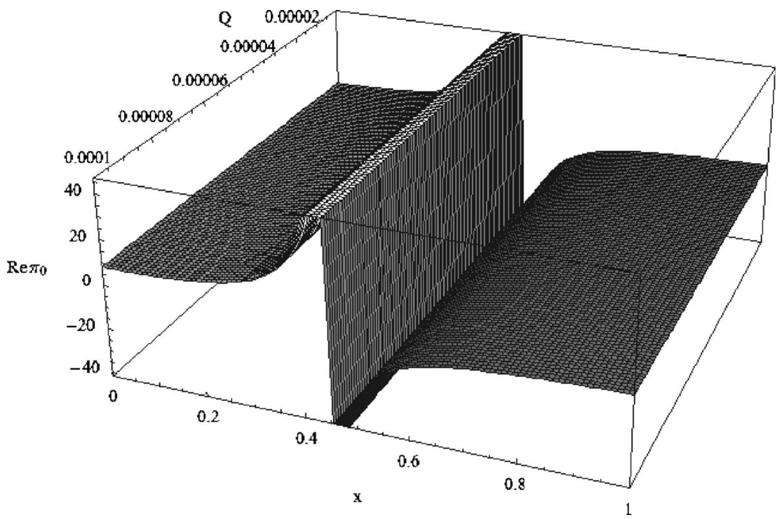

FIG. 2. Re $\pi_{00}(k)$ and the Goldstone mode in the case of Ψ_1.

the type $(X+Y)^2$ stand for $[X(-k)+Y(-k)][X(k)+Y(k)]$. $K_{00}(k)$ and $K_{11}(k)$ are the free fermion (RPA) density-density and current-current correlation functions.[16] In the long-wavelength limit ($k/k_f \ll 1$), they can be explicitly evaluated from the general expressions,[16]

$$K_{00}(k,\omega) = \frac{m}{2\pi}\left[1 - \Theta(x^2-1)\frac{|x|}{\sqrt{x^2-1}} + i\Theta(1-x^2)\frac{|x|}{\sqrt{1-x^2}}\right], \tag{35}$$

$$K_{11}(k,\omega) = \frac{2n_f}{m}\left[-x^2 - \frac{k^2}{24\pi n_f} + \Theta(x^2-1)|x|\sqrt{x^2-1} + i\Theta(1-x^2)|x|\sqrt{1-x^2}\right], \tag{36}$$

where $x=\frac{m\omega}{k_f k}$, k_f the Fermi wave vector, n_f the fermion density, and Θ the Heaviside step function. The mass appearing in expressions for K_{00} and K_{11} is equal to the bare electron mass only in the RPA approximation (in which we work here).

Focusing on the charge channel only [Eq. (33)] and integrating out first $\delta\rho_C^B$, then a_0^C and δa_C, we arrive at the density-density correlator,

$$\pi_{00}(k) = \frac{\left(\frac{k}{2\pi}\right)^2}{\frac{2\bar{\rho}_b}{m} - 2K_{11} + V_C\left(\frac{k}{2\pi}\right)^2 - \frac{\left(\frac{k}{2\pi}\right)^2}{\frac{2\bar{\rho}_b k^2}{m\omega^2} - 2K_{00}}}. \tag{37}$$

In the limiting case $x \ll 1$: $K_{00} \approx \frac{m}{2\pi}(1+ix)$ and $K_{11} \approx -\frac{k^2}{12\pi m} + i\frac{2n_f}{m}x$, and we conclude that as $\omega \to 0$ (and then $k \to 0$), the system is incompressible in the charge channel, so long as there is a thermodynamically significant density of bosons $\bar{\rho}_b$.

In the pseudospin channel, we are primarily looking for the signature of a Bose condensate, i.e., whether there exists a Goldstone mode of broken symmetry and what is the long-range order of the state. Therefore, in Eq. (34) we set $A_\mu = B_\mu = 0$ and integrate over a_0^S, a_S, and $\delta\rho_S^B$,

$$\langle \theta_S(-k)\theta_S(k)\rangle = \frac{V_S}{\omega^2 \frac{\frac{1}{2}V_S + \alpha}{\alpha} - \frac{2\bar{\rho}_b V_S}{m}k^2}, \tag{38}$$

where $\alpha = \frac{1}{4}K_{00}^{-1} - K_{11}\left(\frac{2\pi}{k}\right)^2$ (in Appendix, we give the full linear response in the pseudospin channel). Indeed, there exists a Goldstone mode, albeit with a small dissipative term (which, if desired, can be removed by pairing construction[10]),

$$\omega^0(k) = \sqrt{\frac{2\bar{\rho}_b V_S}{m}}k - i\frac{V_S}{16\pi^{3/2}\sqrt{n_f}}k^3. \tag{39}$$

Even for large x, it is easy to check that the pole remains at the same value if we assume $\bar{\rho}_b \gg n_f$ (which is, in fact, the most appropriate assumption in this case). Also, the imaginary term disappears in this case. Such robust Goldstone mode implies the existence of a true ODLRO and the genuine Bose condensate. Goldstone mode $\omega^0(k)$ [Eq. (39)] is easily observed in Fig. 2, where we plotted the real part of density-density correlation function $\pi_{00}(k)$ [Eq. (A1)] in terms of parameters $Q \equiv k/k_f$ and $x \equiv \omega/(kk_f)$. Other (fixed) parameters are $m=l_B=1$, $d=0.5$, $\epsilon=12.6$, $V_S=\pi d/\epsilon$, $\bar{\rho}_b + n_f = 1/(4\pi)$, and $\eta = n_f/\bar{\rho}_b = 1/10$.

Let us return to the second case, that of Eq. (3) and dominant intracorrelations. According to Fig. 1, relations [Eq. (32)] are modified to become

$$\frac{1}{2\pi}\nabla \times \mathbf{a}^{F\sigma} = 2\delta\rho^{F\sigma} + 2\delta\rho^{B\sigma},$$

$$\frac{1}{2\pi}\nabla \times \mathbf{a}^{B\sigma} = 2\delta\rho^{F\sigma} + \delta\rho^{B\uparrow} + \delta\rho^{B\downarrow}. \tag{40}$$

It is obvious that in this case we have only three linearly independent gauge fields, namely, $a_C = \frac{a^{F\uparrow}+a^{F\downarrow}}{2} = \frac{a^{B\uparrow}+a^{B\downarrow}}{2}$, $a_S = \frac{a^{F\uparrow}-a^{F\downarrow}}{2}$, and $a_{FS} = \frac{a^{B\uparrow}-a^{B\downarrow}}{2}$. Introducing the same substitutions

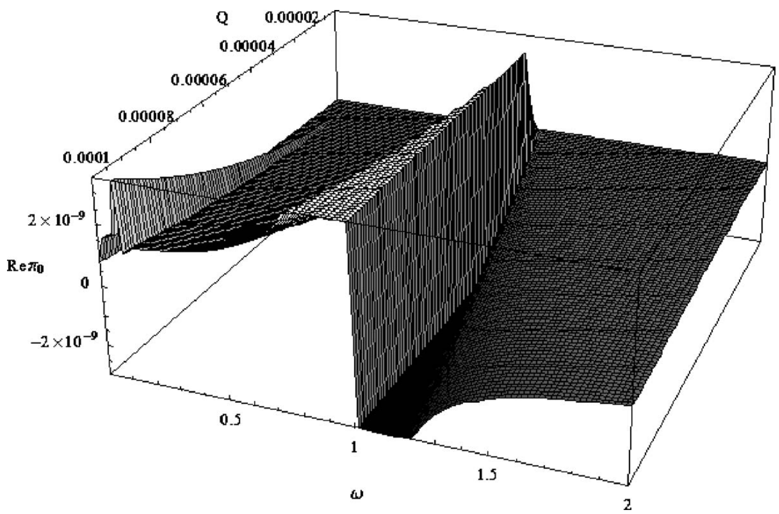

FIG. 3. Re $\pi_{00}(k)$ in the case of Ψ_2.

as before, the Lagrangian again decouples into a charge channel,

$$\mathcal{L}_C = K_{00}(\delta a_0^C)^2 + K_{11}(\delta a_C)^2 + i\omega\delta\rho_C^B\theta_C - \delta\rho_C^B\delta a_0^C - \frac{\bar{\rho}_b}{m}k^2\theta_C^2 - \frac{\bar{\rho}_b}{m}(\delta a_C)^2 + \frac{ik}{2\pi}a_0^C a_C - \frac{1}{2}V_C\left(\frac{k}{2\pi}\right)^2 a_C^2, \quad (41)$$

and a pseudospin channel,

$$\mathcal{L}_{PS} = K_{00}(\delta a_0^S)^2 + K_{11}(\delta a_S)^2 + i\omega\delta\rho_S^B\theta_S - \delta\rho_S^B\delta a_0^{FS} - \frac{\bar{\rho}_b}{m}k^2\theta_S^2 - \frac{\bar{\rho}_b}{m}(\delta a_{FS})^2 + \frac{ik}{2\pi}a_0^S a_{FS} + \frac{ik}{2\pi}a_0^{FS}(a_S - a_{FS}) - \frac{1}{2}V_S\left(\frac{k}{2\pi}\right)^2 a_S^2, \quad (42)$$

where $\delta a_0^{FS} \equiv a_0^{FS} - B_0$ and $\delta a_{FS} = a_{FS} - B$; all the other symbols have retained their meanings.

This time we will not analyze the charge channel in detail. To this end, we note that the system in incompressible in this sector, the fact which is easily established by integrating out all the gauge fields, densities, and boson phase in Eq. (41).

In the pseudospin channel, a calculation of the density-density correlator leads to the conclusion that in this channel, the system is compressible (see also Fig. 3). The θ-θ correlator is

$$\langle\theta_S(-k)\theta_S(k)\rangle = \frac{\frac{1}{k^2}\beta\gamma}{\left(\frac{\omega}{2\pi}\right)^2(\beta+\gamma) - \frac{2\bar{\rho}_b}{m}\beta\gamma}, \quad (43)$$

where $\beta = \frac{1}{2K_{00}}\left(\frac{k}{2\pi}\right)^2 + \frac{2\bar{\rho}_b}{m}$ and $\gamma = V_S\left(\frac{k}{2\pi}\right)^2 - 2K_{11}$. For small k/k_f and x, the correlator diverges for $\omega^0 = \frac{4\pi\bar{\rho}_b}{m} = \text{const}$, which obviously contradicts the original assumption for the range of x and hence we reject this pole. For $x \gg 1$ (and still $k \ll k_f$), the relations [Eqs. (35) and (36)] are approximately $K_{00} \approx -\frac{1}{4\pi x^2}$ and $K_{11} \approx -\frac{n_f}{m}$, and we obtain two poles,

$$\omega^0(k) = \frac{4\pi n_f}{m}\sqrt{\frac{1}{2} + \eta - \frac{1}{2}\sqrt{1+4\eta}}, \quad (44)$$

$$\Omega^0(k) = \frac{4\pi n_f}{m}\sqrt{\frac{1}{2} + \eta + \frac{1}{2}\sqrt{1+4\eta}}, \quad (45)$$

where $\eta = \bar{\rho}_b/n_f$ is the ratio of boson to fermion density [Eqs. (44) and (45) hold for any η, although in the physical limit that we are presently interested, η may be regarded as small]. In Fig. 3 we plotted the real part of the density-density correlation function in the case of Ψ_2 [Eq. (A4)]. In contrast to Fig. 2, here we opt for ω and Q as free parameters and set $d=1.5$ and $\eta = \bar{\rho}_b/n_f = 1/10$ as the more likely values in this case. Distinctive feature of Fig. 3 at $\omega \cong 1$ is the plasma frequency Ω^0 and the smaller singularity at $\omega \cong 1/10$ corresponds to ω^0. There is also a striking absence of Goldstone mode in this case.

We now proceed to calculate ODLRO in the pseudospin channel of Ψ_2. As it turns out, ODLRO will be nontrivially modified and assume algebraic form. We know that interaction does not affect the value of characteristic exponent[19] and therefore set $V_S \equiv 0$. Bearing in mind that we work in the long-wavelength limit, we arrive at the following expression for the correlator:

$$\langle\theta_S(-k)\theta_S(k)\rangle = \frac{(2\pi\omega_P/k^2)(\omega^2 - \omega_P^2\eta)}{\{\omega^2 - [\omega^0(k)]^2\}\{\omega^2 - [\Omega^0(k)]^2\}}, \quad (46)$$

where we introduced $\omega_P = \frac{4\pi n_f}{m}$. After contour integration over ω,[19]

$$\langle\theta_S(-k)\theta_S(k)\rangle = -\frac{2\pi}{k^2}f(\eta),$$

where $f(\eta) = \frac{1}{\sqrt{1+4\eta}}$, which leads to the algebraic ODLRO,

$$\langle e^{i\theta_S(\mathbf{r})}e^{-i\theta_S(\mathbf{r}')}\rangle \propto \frac{1}{|\mathbf{r}-\mathbf{r}'|^{f(\eta)}} \approx |\mathbf{r}-\mathbf{r}'|^{-[1-2\eta+o(\eta^2)]}. \quad (47)$$

This algebraic ODLRO persists as long as $\eta > 0$ (function f is positive everywhere in this domain). The expression Eq. (47) is formally reminiscent of BKT XY ordering; only the role of the temperature is overtaken by the parameter η (the analysis of this paper assumes temperature $T=0$). Pursuing this analogy further, we conclude that the relative fluctuations of composite boson and composite fermion densities represent the mechanism which may lead to the ultimate breakdown of the 111 condensate.

IV. EVOLUTION OF THE GROUND STATE WITH d

In order to investigate the transition from the incompressible, 111-like state at lower d/l_B, to the compressible, possibly vortex metal-like state at higher d/l_B, we are motivated to introduce what we call *generalized vortex metal*. In addition to the ordinary vortex metal (Ψ_2), we include (in each layer) another kind of composite fermions that connect to the composite boson sea as in the case of Ψ_1. The generalized vortex metal is clearly the only additional option left of connecting electrons divided in composite bosons and composite fermions beside the two extreme cases, Ψ_1 and Ψ_2. Again, in this state some of composite fermions connect in the manner of 111 state to the composite bosons and the rest of composite fermions connect exclusively to the composite bosons of the same layer in the manner of the Rezayi-Read state. This is succinctly represented by the following gauge field constraints:

$$\frac{1}{2\pi}\nabla\times a^{B\sigma}=\delta\rho_{B\uparrow}+\delta\rho_{B\downarrow}+\delta\rho_{F\uparrow}^{(1)}+\delta\rho_{F\downarrow}^{(1)}+2\delta\rho_{F\sigma}^{(2)}, \tag{48}$$

$$\frac{1}{2\pi}\nabla\times a_1^{F\sigma}=\delta\rho_{B\uparrow}+\delta\rho_{B\downarrow}+2\delta\rho_{F\sigma}^{(1)}+2\delta\rho_{F\sigma}^{(2)}, \tag{49}$$

$$\frac{1}{2\pi}\nabla\times a_2^{F\sigma}=2\delta\rho_{B\sigma}+2\delta\rho_{F\sigma}^{(1)}+2\delta\rho_{F\sigma}^{(2)}, \tag{50}$$

where the superscripts (1) and (2) indicate composite fermion species in each layer. Chern-Simons theory easily follows from the above gauge field equations and yields incompressible behavior in the charge channel. In the pseudospin channel,

$$\begin{aligned}\mathcal{L}_{PS}={}&K_{00}^{(1)}(\delta a_{0,1}^{FS})^2+K_{00}^{(2)}(\delta a_{0,2}^{FS})^2+K_{11}^{(1)}(\delta a_1^{FS})^2+K_{11}^{(2)}(\delta a_2^{FS})^2\\&+i\omega\delta\rho_S^B\theta_S-\delta\rho_S^B\delta a_0^S-\frac{\bar{\rho}_b}{m}k^2\theta_S^2-\frac{\bar{\rho}_b}{m}(\delta a_S)^2\\&+\frac{ik}{2\pi}a_{0,1}^{FS}(a_1^{FS}-a_S)+\frac{ik}{2\pi}a_0^S(a_2^{FS}-a_1^{FS})+\frac{ik}{2\pi}a_{0,2}^{FS}a_S\\&-\frac{1}{2}V_S\left(\frac{k}{2\pi}\right)^2|a_2^{FS}|^2-\frac{1}{2}V_{hc}\left(\frac{k}{2\pi}\right)^2a_S(a_1^{FS}-a_S),\end{aligned} \tag{51}$$

where the linearly independent fields are given by $a_S=\frac{a^{B\uparrow}-a^{B\downarrow}}{2}$, $a_1^{FS}=\frac{a_1^{F\uparrow}-a_1^{F\downarrow}}{2}$, and $a_2^{FS}=\frac{a_2^{F\uparrow}-a_2^{F\downarrow}}{2}$, subscripts 1 and 2 distinguish between composite fermion species and S denotes antisymmetric combination of the densities in two layers (like in Sec. III). A noteworthy feature of the Lagrangian [Eq. (51)] is the existence of V_{hc}, the hard-core repulsion term between the two species of composite fermions inside each layer. The presence of such a term (added by hand) is natural if we imagine composite fermions residing in two separate Fermi spheres. However, the danger of blindly introducing this term is that it may incidentally bring about the incompressible behavior (otherwise not present) in the system. We have verified that this is *not* the case here; i.e., the system remains incompressible whether or not we choose to introduce V_{hc}. It therefore appears more intuitive to keep V_{hc}, taking the limit $V_{hc}\to\infty$ in the end. Step by step, eliminating all the gauge fields, we are lead to the following correlation function:

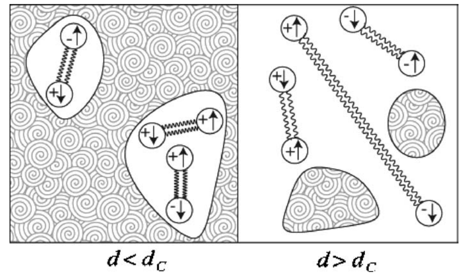

FIG. 4. Evolution of the ground state with varying d, before and after the transition at $d=d_C$. The regions with meron pairs represent the vortex metal (Ψ_2) phase. The background represents the superfluid (Ψ_1) phase.

$$\langle\theta_S(-k)\theta_S(k)\rangle=\frac{V_S+\dfrac{2n_{f2}}{m}\left(\dfrac{2\pi}{k}\right)^2}{\omega^2-\left[\dfrac{2\bar{\rho}_bV_S}{m}k^2+\left(\dfrac{4\pi}{m}\right)^2\bar{\rho}_bn_{f2}\right]}, \tag{52}$$

and the low-energy spectrum is dominated by the plasma frequency,

$$\omega^0(k)=\frac{4\pi}{m}\sqrt{\bar{\rho}_bn_{f2}}, \tag{53}$$

where n_{f2} is the density of the composite fermions which bind exclusively within the layer they belong. Generalized vortex metal therefore is a state that only supports gapped collective excitations, despite the presence of composite bosons and the kind of composite fermions which enforce interlayer correlation. If it is pertinent to the region of the tunneling experiments of Spielman *et al.*[5] and counterflow experiments of Kellogg *et al.*,[6] we believe that our homogeneous theory of Secs. III and IV then suggests that (generalized) vortex metal can appear only as localized islands (due to presence of disorder at low temperatures) amidst the background of Ψ_1 phase (Fig. 4). In Fig. 4 weakly coupled vortex-antivortex pairs are depicted, i.e., meron-antimeron pairs (due to the charge degree of freedom, there are four kinds of merons[2]) inside the vortex metal phase. They are expected to exist in the vortex-metal phase on the grounds of disordering of the correlated phase. As argued in Ref. 10, the inclusion of composite fermions into the 111 state (Ψ_1 and Ψ_2) corresponds to the creation of meron-antimeron pairs. There are more pairs and more of larger size as d increases consistently with the BKT picture of the phase that supports algebraic ODLRO [Eq. (47)].

V. FURTHER COMPARISON WITH EXPERIMENTS

In this section, we wish to address in depth the potential of the model states, Ψ_1 and Ψ_2, in explaining the phenomenology of experiments on bilayer. The key question in this analysis is: what is the nature of the compressible phase corresponding to higher d/l_B that still harbors some of the intercorrelation present at lower d/l_B?[9]

The answer to this question cannot be given by looking at simple transport properties. In Sec. II, it was shown that both Ψ_1 and Ψ_2 in certain regimes can recover the two main experimental findings of Kellogg *et al.* in drag experiments: the semicircle law[9] and the quantization of Hall drag resistance.[20] On the other hand, our Chern-Simons RPA approach at $T=0$ stresses that all that states considered in this paper are incompressible. However, at finite T, a finite energy[10] is needed to excite a meron in Ψ_2 and therefore Ψ_2 seems like a better candidate for exhibiting compressible behavior at any finite T or, at least, a very small gap. Furthermore, within the vortex metal picture, Ψ_2 allows the following simple scenario. For $\rho_{bxx} \ll \rho_{fxx} \ll \delta$, one gets the semicircle law derived in Sec. II. As d/l_B increases, the density of bosons decreases and one enters the regime $\rho_{fxx} \ll \rho_{bxx} \ll \delta$, where $|\rho_{xx}^D| \gg \rho_{xy}^D$ (as witnessed in the experiments[9]). The persistence of enhanced longitudinal drag resistance[9] up to very high d/l_B provides additional support to our choice of Ψ_2 which can explain the remaining intercorrelation (drag) in the case where explicit tunneling is absent. Finally, as $\rho_{bxx} \to \infty$, both resistances go to zero, the bilayer decouples, and bosons vanish from the system.

Our picture is certainly incomplete because it does not explicitly include the effects of disorder (which must be very relevant for the physics of bilayer in the regimes $d \sim l_B$—a simple way to see this is to look at the behavior of measured counterflow resistances[6,7] ρ_{xx}^{CF} and ρ_{xy}^{CF} that enter the insulating regime very quickly after passing through $\nu_T=1$). Fertig and Murthy[21] provided a realistic model for the effects of disorder and in their disorder-induced coherence network in the incompressible phase of the bilayer, merons are able to sweep by hopping across the system, causing the activated behavior of resistance (dissipation) in counterflow. This finding is consistent with our own.

At the end our picture is in the spirit of the Stern and Halperin proposal[13] but instead of the 1/2 compressible phase coexisting in a phase separated picture with the superfluid phase (Ψ_1), we assume the existence of the vortex metal phase (Ψ_2). This coincides with the proposal of Fertig and Murthy[21] for the incompressible region that explains the "imperfect" superfluid behavior. It is the continuous extrapolation of this phase separated picture that brings and favors Ψ_2 for larger d/l_B (instead of Ψ_1). There Ψ_2 is able to explain the persistence of intercorrelations through enhanced longitudinal drag accompanied by the absence of tunneling and phase coherence.[20]

Finally, we are able to account for the effects of the layer density imbalance in tunneling, drag,[22] and counterflow[23] experiments. Spielman *et al.*[22] observed that small density imbalance stabilizes the resonant tunneling peak—a simple reason for this is that Ψ_1 can easily accommodate the fluctuations in density [see comment after Eq. (2)]. Because of the same reason, Hall drag resistance remains quantized up to larger d/l_B in the presence of density imbalance. On the other hand, the enhancement of longitudinal drag resistance at large d/l_B was also reported[9] to be insensitive to density imbalance. While the reason for this cannot be seen only from looking at the form of Ψ_2 [this state constrains both fermion and boson numbers in two layers, see comment after Eq. (4)], we believe that meron excitations are responsible for absorbing the density fluctuations, especially at finite T.

Recently, the quantum Hall bilayer was probed using resonant Rayleigh scattering[24] for samples with different tunneling amplitudes and when the in-plane magnetic field is present. They detected a nonuniform spatial structure in the vicinity of the transition, suggesting a phase-separated version of the ground state. Our results (for zero tunneling limit and excluding disorder) hint that such phase separation may indeed be necessary to invoke in order to achieve a full description of the strongly coupled, incompressible phase and the transition in a bilayer.

VI. DISCUSSION AND CONCLUSION

In conclusion, we showed how two model states, Ψ_1 and Ψ_2, can account for the basic phenomenology of the bilayer that came up from various experiments.

A very interesting question pertains to the model state Ψ_2. Effectively the state represents a collection of meron excitations interacting through topological interactions. A question comes when they are in a confined (dipole) phase and when in a metallic (plasma) phase. So in principle we can expect that the static correlator in Eq. (47) can be reproduced by considering a two-dimensional (2D) bosonic model with meron excitations interacting via 2D Coulomb plasma interaction.[25] Therefore we believe that the Laughlin ansatz[26] of considering (static) ground-state correlators as statistical models in 2D can also be applied here. We expect that the ground-state correlators in a dual approach, in which we switch from composite fermion to meron coordinates, can be mapped to a partition function of a 2D Coulomb plasma.[27] The 2D Coulomb plasma has two different phases. For large β (inverse T), the charges form dipoles and the system is with long-range correlations (no mass gap). At some critical β, dissociation of dipoles occurs and we have a plasma phase with a Debye screening, and therefore a mass gap. Thus, calculations that will capture more of the meron contribution than our RPA approach in the Chern-Simons theory may find a transition and exponential decay of the correlator [Eq. (47)] before reaching the $\bar{\rho}_b=0$ limit. Indeed, our ODLRO exponent in Eq. (47) at $\bar{\rho}_b=0$ is 1 which is well above the exponent of the BKT transition or critical exponent 1/4. At that point our system may develop a gap in the pseudospin channel and completely lose interlayer coherence (exponential decay of correlators). Furthermore, we expect that the superfluid portion of the composite boson density will disappear leading to compressible behavior in the charge channel.[19]

This is all consistent with experiments[9,20] which find that, at $d/l_B \approx 1.84$, the vanishing of the conventional quantum Hall effect and the system's Josephson-like tunneling characteristics occur simultaneously. Intercorrelated bosons continue to exist without a superfluid property and lead to enhanced $\nu = 1$ drag at large d/l_B. They disappear from the system around $d/l_B \approx 2.6$.[9]

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APPENDIX

In order to extract the response functions in functional integral formalism, one needs to integrate over all degrees of freedom except those of the external fields. The integration of these fields in the RPA proverbially reduces to the Gaussian integral,

$$\int d(z,z*)\exp(-z*wz+u*z+vz*) = \frac{\pi}{w}\exp\left(\frac{u*v}{w}\right), \quad \mathrm{Re}\, w > 0.$$

For the pseudospin channel in the case of Ψ_1 [Eq. (1)], we therefore obtain the following linear response:

$$\pi_{00}(k) = \frac{1}{\Xi}[\Omega(3V_S + 2\alpha) + 1], \tag{A1}$$

$$\pi_{01}(k) = \pi_{10}(k) = \frac{1}{\Xi}\frac{-i\pi}{k}K_{11}(1+\Omega V_S), \tag{A2}$$

$$\pi_{11}(k) = \frac{1}{\Xi}\left[2V_S(1-\Omega V_S)\left(K_{11} - \frac{\bar{\rho}_b}{m}\right) + K_{11}/K_{00} - 4\alpha\frac{\bar{\rho}_b}{m}\right], \tag{A3}$$

where $\alpha \equiv \frac{1}{4}K_{00}^{-1} - K_{11}(\frac{2\pi}{k})^2$, $\Omega \equiv \left[V_S - \frac{m\omega^2}{2\bar{\rho}_b k^2}\right]^{-1}$, and $\Xi = V_S(1-\Omega V_S) + 2\alpha$.

The response functions in the case of the pseudospin channel of Ψ_2 [Eq. (3)] are

$$\pi_{00} = \frac{1}{\Delta}\left(\frac{k}{2\pi}\right)^2, \tag{A4}$$

$$\pi_{01} = \pi_{10} = \frac{1}{\Delta}\frac{ik}{2\pi}\Lambda, \tag{A5}$$

$$\pi_{11} = \frac{1}{\Delta}\left\{\Lambda^2 + \Delta\left[2K_{11} - \frac{2\bar{\rho}_b}{m} - 16W^4\left(\frac{2\pi\bar{\rho}_b}{m\omega}\right)^4\right]\right\}, \tag{A6}$$

where $W^4 \equiv -\frac{\left(\frac{m\omega^2}{2\bar{\rho}_b(2\pi)^2}\right)^2}{\frac{1}{2K_{00}}(\frac{k}{2\pi})^2 - \frac{m\omega^2}{2\bar{\rho}_b(2\pi)^2} + \frac{2\bar{\rho}_b}{m}}$, $\Delta \equiv W^4 - \frac{m\omega^2}{2\bar{\rho}_b(2\pi)^2} - 2K_{11} + V_S(\frac{k}{2\pi})^2$, and $\Lambda \equiv 4W^4(\frac{2\pi\bar{\rho}_b}{m\omega})^2 - 2K_{11}$.

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